Stochastic DEA model with undesirable outputs: An application to the prediction of anti-HIV therapy efficiency*

BIAN Fuping[1,2**] and TANG Xiaoqin[1,2]

(1. Department of Mathematics, School of Science, Tianjin University, Tianjin 300072, China; 2. Liuhui Institute of Applied Mathematics, Nankai University and Tianjin University, Tianjin 300072, China)

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Abstract This paper proposes a stochastic prediction DEA model with undesirable outputs and simplifies the process using chance-constrained techniques in order to obtain an equivalent linear programming formulation. The existence and stability of the optimal solutions have been proved. And the model is used to describe and predict the efficiency of anti-HIV therapy in AIDS patients.

Keywords: data envelopment analysis (DEA), decision making units (DMU), undesirable outputs, stochastic DEA model, anti-HIV therapy.

Acquired immunity deficiency syndrome (AIDS), caused by the human immunodeficiency virus (HIV), is one of the most serious epidemic diseases that infected an estimated 5 million people worldwide last year and the number is still relentlessly increasing. Once HIV enters the body, the human immune system tries to get rid of it. However, anti-HIV therapy is a long term process and is very expensive together with some side effects. By far, many researchers have developed some methods, for example, using non-linear dynamic models[1,2] which describe the interaction of HIV and T-cells in the immune system to deal with the data in efficiency evaluation of anti-HIV therapy. This work aims at finding efficient strategies for prediction of the efficiency of the anti-HIV therapy.

DEA, originally introduced by Charnes et al. measures the relative efficiencies among the decision making units (DMUs) with multiple inputs and multiple outputs as a linear programming formulation and has become a standard non-parametric approach to productivity analysis[3]. The prevailing use of DEA has been mainly applied to performance evaluation on the basis of an anterior and exact data set, while inputs and outputs of DMUs are actually ever-changeful and unexpected. As a result, DEA efficiency measurement may be sensitive to such stochastic variations. In order to incorporate stochastic inputs and outputs into conventional DEA models, Land et al.[4] and Bian et al.[5,6] have proposed the chance-constrained models and tried to take randomness into consideration. However, all the stochastic models have a problem in the computational feasibility due to the existence of non-linear constraints. To solve this problem, we have to consider some special distributions such as a normal distribution and so on, and simplify the process. Here, we propose a new type of stochastic DEA model to predict the efficiency of anti-HIV therapy which may provide helpful information and optimal strategies to both patients and doctors.

1 Stochastic DEA model with undesirable outputs

1.1 Construction of the stochastic DEA model with undesirable outputs

Let $x_j = (x_{1j}, \cdots, x_{mj})^{\mathrm{T}}$ be $(m \times 1)$ determined input vectors and $y_j = (y_{1j}, \cdots, y_{sj})^{\mathrm{T}}$ be $(s \times 1)$ determined output vectors. Then the model with undesirable outputs is

$$\max\Big(\sum_{r=1}^{p}\mu_r y_{r0} - \sum_{r=p+1}^{s}\mu_r y_{r0}\Big) + \mu_0$$

$$\text{(M1) s.t.}\begin{cases}\Big(\sum_{r=1}^{p}\mu_r y_{rj} - \sum_{r=p+1}^{s}\mu_r y_{rj}\Big) + \mu_0 \leqslant \sum_{i=1}^{m}\omega_i x_{ij}\\ \sum_{i=1}^{m}\omega_i x_{i0} = 1\\ \omega \geqslant 0, \mu \geqslant 0\\ i = 1,2,\cdots,m\\ r = 1,2,\cdots,s\\ j = 1,2,\cdots,n.\end{cases}$$

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** To whom correspondence should be addressed. E-mail: fpbian@tju.edu.cn

Let us consider all outputs to be jointly normally distributed in the following chance constrained version of a stochastic DEA model:

$$\max\Big(\sum_{r=1}^{p}\mu_r\hat{y}_{r0}-\sum_{r=p+1}^{s}\mu_r\hat{y}_{r0}\Big)+\mu_0$$

$$(\mathrm{M2})\ \text{s.t.}\begin{cases} P\Big\{\sum_{i=1}^{m}\omega_i x_{ij}-\Big(\sum_{r=1}^{p}\mu_r\hat{y}_{rj}-\sum_{r=p+1}^{s}\mu_r\hat{y}_{rj}\Big)-\mu_0\geqslant 0\Big\} \\ \quad\geqslant 1-\alpha \\ \sum_{i=1}^{m}\omega_i x_{i0}=1 \\ \omega\geqslant 0,\ \mu\geqslant 0 \\ i=1,2,\cdots,m \\ r=1,2,\cdots,s \\ j=1,2,\cdots,n, \end{cases} \tag{1}$$

where $\hat{y}_j=(\hat{y}_{1j},\cdots,\hat{y}_{sj})^{\mathrm{T}}$ present $(s\times 1)$ random output vectors, and $\bar{y}_j=(\bar{y}_{1j},\cdots,\bar{y}_{sj})^{\mathrm{T}}$ stands for the corresponding vectors of expected values of output for each DMU, P means "probability" and α is a determined number between 0 and 1 which represents the risk level.

1.2 Reformulation of stochastic DEA model with chance-constrained techniques

It can be easily thought that the above stochastic DEA model (M2) needs to be reformulated to obtain its computational feasibility. In this study, the constraints and objective of (M2) are formulated by methods presented in Refs. [4, 5], which have given the details of how to incorporate the Chance Constrained Programming (CCP) techniques into the DEA models.

Let

$$\hat{z}_{rj}=\begin{cases}\hat{y}_{rj}, & r=1,2,\cdots,p\\ -\hat{y}_{rj}, & r=p+1,\cdots,s,\end{cases}$$

the constraints of (M2) can be rewritten as follows:

$$P\Big\{\sum_{r=1}^{s}\mu_r\hat{z}_{rj}\leqslant\sum_{i=1}^{m}\omega_i x_{ij}-\mu_0\Big\}\geqslant 1-\alpha. \tag{2}$$

Eq. (2) is equivalent to

$$P\left\{\frac{\sum_{r=1}^{s}\mu_r(\hat{z}_{rj}-\bar{z}_{rj})}{\sqrt{\mathrm{Var}_j}}\leqslant\frac{\sum_{i=1}^{m}\omega_i x_{ij}-\sum_{r=1}^{s}\mu_r\bar{z}_{rj}-\mu_0}{\sqrt{\mathrm{Var}_j}}\right\}\geqslant 1-\alpha, \tag{3}$$

where $\bar{z}_{rj}$ is the expected value of $\hat{z}_{rj}$ and

$$\mathrm{Var}_j=(\mu_1,\mu_2,\cdots,\mu_s)\times\begin{pmatrix} v(\hat{z}_{1j}) & \mathrm{Cov}(\hat{z}_{1j},\hat{z}_{2j}) & \cdots & \mathrm{Cov}(\hat{z}_{1j},\hat{z}_{sj})\\ \mathrm{Cov}(\hat{z}_{2j},\hat{z}_{1j}) & v(\hat{z}_{2j}) & \cdots & \mathrm{Cov}(\hat{z}_{2j},\hat{z}_{sj})\\ \vdots & \vdots & \cdots & \vdots\\ \mathrm{Cov}(\hat{z}_{sj},\hat{z}_{1j}) & \mathrm{Cov}(\hat{z}_{sj},\hat{z}_{2j}) & \cdots & v(\hat{z}_{sj})\end{pmatrix}\times\begin{pmatrix}\mu_1\\ \mu_2\\ \vdots\\ \mu_s\end{pmatrix},$$

where Var_j indicates the variance-covariance matrix of the rth output, in which the symbol "v" stands for a variance and the symbol "Cov" refers to a covariance operator.

To reformulate Eq. (3) by CCP, we introduce a new variable (q_{rj}).

Let

$$q_j=\frac{\sum_{r=1}^{s}v_r(\hat{z}_{rj}-\bar{z}_{rj})}{\sqrt{\mathrm{Var}_j}},$$

which follows the standard normal distribution with zero mean and unit variance.

Then

$$P\left\{q_j\leqslant\frac{\sum_{i=1}^{m}\omega_i x_{ij}-\sum_{r=1}^{s}\mu_r\bar{z}_{rj}-\mu_0}{\sqrt{\mathrm{Var}_j}}\right\}\geqslant 1-\alpha. \tag{4}$$

Since q_j follows the standard normal distribution, the invertibility of Eq. (4) is executed as

$$\frac{\sum_{i=1}^{m}\omega_i x_{ij}-\sum_{r=1}^{s}\mu_r\bar{z}_{rj}-\mu_0}{\sqrt{\mathrm{Var}_j}}\geqslant F^{-1}(1-\alpha),$$

in which F stands for a cumulative distribution function of the normal distribution and F^{-1} indicates its inverse function. That is,

$$\sqrt{\mathrm{Var}_j}F^{-1}(1-\alpha)+\sum_{r=1}^{s}\mu_j\bar{z}_{rj}\leqslant\sum_{i=1}^{m}\omega_i x_{ij}-\mu_0. \tag{5}$$

So, the DEA stochastic model with undesirable outputs is obtained by replacing Eq. (1) by Eq. (5), and its resulting formulation becomes

$$\max E\Big(\sum_{r=1}^{p}\mu_r\hat{y}_{r0}-\sum_{r=p+1}^{s}\mu_r\hat{y}_{r0}\Big)+\mu_0$$

$$(\mathrm{M3})\ \text{s.t.}\begin{cases}\sum_{i=1}^{m}\omega_i x_{ij}-\sum_{r=1}^{p}\mu_r\bar{z}_{rj}-\mu_0\\ \quad\geqslant\sqrt{\mathrm{Var}_j}F^{-1}(1-\alpha)\\ \sum_{i=1}^{m}\omega_i x_{i0}=1\\ \omega\geqslant 0,\ \mu\geqslant 0\\ i=1,2,\cdots,m\\ r=1,2,\cdots,s\\ j=1,2,\cdots,n.\end{cases}$$

However, (M3) still has a computational difficulty as the constraints include the variance Var_j formulated by a quadratic expression. To obtain a linear programming equivalent to (M3), we have to assume that a stochastic variable $\hat{y}_{rj}$ of each output is expressed by $\hat{y}_{rj} = \bar{y}_{rj} + b_{rj}\xi (r = 1, \cdots, s, j = 1, \cdots, n)$, where $\bar{y}_{rj}$ is the expected value of $\hat{y}_{rj}$ and b_{rj} is its standard deviation. We also assume that the single random variable ξ follows a normal distribution $N(0, \sigma^2)$.

Using such assumptions, Var_j becomes

$$\mathrm{Var}_j = (\mu_1, \mu_2, \cdots, \mu_s) \times \begin{pmatrix} b_{1j}^2\sigma^2 & b_{1j}b_{2j}\sigma^2 & \cdots & -b_{1j}b_{sj}\sigma^2 \\ b_{2j}b_{1j}\sigma^2 & b_{2j}^2\sigma^2 & \cdots & -b_{2j}b_{sj}\sigma^2 \\ \vdots & \vdots & \vdots & \vdots \\ -b_{sj}b_{1j}\sigma^2 & -b_{sj}b_{2j}\sigma^2 & \cdots & b_{sj}^2\sigma^2 \end{pmatrix} \times \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_s \end{pmatrix} = \Big(\sum_{r=1}^{p} \mu_j b_{rj}\sigma - \sum_{r=p+1}^{s} \mu_j b_{rj}\sigma \Big)^2. \quad (6)$$

Substituting Eq. (6) into (M3) provides

$$\max E\Big(\sum_{r=1}^{p} \mu_r \hat{y}_{r0} - \sum_{r=p+1}^{s} \mu_r \hat{y}_{r0} \Big) + \mu_0 = V_p$$

$$(\mathrm{M4})\ \mathrm{s.t.}\begin{cases} \sum_{r=1}^{p} \mu_j [b_{rj}\sigma F^{-1}(1-\alpha) + \bar{y}_{rj}] - \sum_{r=p+1}^{s} \mu_j [b_{rj}\sigma F^{-1}(1-\alpha) + \bar{y}_{rj}] + \mu_0 \\ \leqslant \sum_{i=1}^{m} \omega_i x_{ij} \\ \sum_{i=1}^{m} \omega_i x_{i0} = 1 \\ \omega \geqslant 0, \mu \geqslant 0 \\ i = 1, 2, \cdots, m \\ r = 1, 2, \cdots, s \\ j = 1, 2, \cdots, n. \end{cases}$$

Theorem 1. (M4) exists a feasible solution, and the optimal value V_p satisfies $V_p \leqslant 1$.

Proof. Let

$$\omega^* = \frac{x_0}{\| x_0 \|^2}, \quad \mu^* = (\mu_1^*, 0, \cdots, 0)^{\mathrm{T}} \in R^s,$$

here,

$$\mu_1^* = \min_{1 \leqslant j \leqslant n} \frac{\omega^{*\mathrm{T}} x_j - \mu_0}{y_{1j} + b_{1j}\sigma F^{-1}(1-\alpha)} > 0,$$

then

$$\omega^* \geqslant 0, \mu^* \geqslant 0, \quad \omega^* x_0 = \frac{x_0^{\mathrm{T}}}{\| x_0 \|^2} x_0 = 1,$$

$$\sum_{i=1}^{m} \omega_i^* x_{ij} - \Big[\sum_{r=1}^{p} \mu_j^* (b_{rj}\sigma F^{-1}(1-\alpha) + \bar{y}_{rj}) - \sum_{r=p+1}^{s} \mu_j^* (b_{rj}\sigma F^{-1}(1-\alpha) + \bar{y}_{rj}) \Big] - \mu_0$$

$$= \sum_{i=1}^{m} \omega_i^* x_{ij} - [\mu_1^* (b_{1j}\sigma F^{-1}(1-\alpha) + \bar{y}_{1j})] - \mu_0 \geqslant 0.$$

Therefore, ω^*, μ^* is the feasible solution of (M4).

The objective function of the j_0th DMU of (M1) is replaced by the expected value $E\big(\sum_{r=1}^{p} \mu_r \hat{y}_{r0} - \sum_{r=p+1}^{s} \mu_r \hat{y}_{r0} \big)$ in (M4). Similarly, $\sum_{r=1}^{p} \mu_r \hat{y}_{r0} - \sum_{r=p+1}^{s} \mu_r \hat{y}_{r0}$ on the left-hand side of (M1) is replaced by

$$\sum_{r=1}^{p} \mu_j^* (b_{rj}\sigma F^{-1}(1-\alpha) + \bar{y}_{rj}) - \sum_{r=p+1}^{s} \mu_j^* (b_{rj}\sigma F^{-1}(1-\alpha) + \bar{y}_{rj})$$

in (M4). These two changes imply that since each output is a stochastic variable, the output configuration influencing a shape of the efficiency frontier depends upon how much the output deviates from its average $\bar{y}_{rj}$. Such a deviation is measured by $b_{rj}\sigma F^{-1}(1-\alpha)$ in (M4). According to the dual theory, (M4) becomes

$$\min\theta$$

$$(\mathrm{M5})\ \mathrm{s.t.}\begin{cases} \sum_{j=1}^{n} \lambda_j x_{ij} + S_{IP} = \theta x_{i0}, \quad i = 1, 2, \cdots, m \\ \sum_{j=1}^{n} \lambda_j [\bar{y}_{lj} + b_{lj}\sigma F^{-1}(1-\alpha)] - S_{PR} = \bar{y}_{l0}, \quad l = 1, 2, \cdots, p \\ \sum_{j=1}^{n} \lambda_j [\bar{y}_{hj} + b_{hj}\sigma F^{-1}(1-\alpha)] + S_{NR} = \bar{y}_{h0}, \quad h = p+1, \cdots, s \\ \sum_{j=1}^{n} \lambda_j = 1 \\ S_{IP}, S_{PR}, S_{NR} \geqslant 0 \\ \lambda_j \geqslant 0, \quad j = 1, 2, \cdots, n. \end{cases}$$

Definition 1. DMU_{j0} is stochastically DEA efficient to the indicated levels of probability if and only if the smallest optimal value of the problem in (M5)

is 1. Otherwise DMU_{j0} is inefficient.

1.3 Extension of the stochastic DEA model with undesirable outputs

In the above models, we only consider the situation that all the inputs and outputs change radically, but in practice, both may alter differently. Therefore, it is reasonable for us to think about the nonradical case[7], then the programming is

$$\min \frac{1}{m}\sum_{i=1}^{m}\theta_i + \frac{1}{s-p}\sum_{h=p+1}^{s}\beta_h - \frac{1}{p}\sum_{l=1}^{p}\beta_l = V$$

$$\text{(M6)s.t.}\begin{cases}\sum_{j=1}^{n}\lambda_j x_{ij} \leqslant \theta x_{i0}, & i = 1,2,\cdots,m\\ \sum_{j=1}^{n}\lambda_j[\bar{y}_{lj} + b_{lj}\sigma F^{-1}(1-\alpha)] \geqslant \beta_l \bar{y}_{l0}, & l = 1,2,\cdots,p\\ \sum_{j=1}^{n}\lambda_j[\bar{y}_{hj} + b_{hj}\sigma F^{-1}(1-\alpha)] \leqslant \beta_h \bar{y}_{h0}, & h = p+1,\cdots,s\\ \sum_{j=1}^{n}\lambda_j = 1\\ 0 \leqslant \theta_i \leqslant 1, & i = 1,2,\cdots,m\\ 0 \leqslant \beta_h \leqslant 1, & h = p+1,\cdots,s\\ \beta_l \geqslant 1, & l = 1,2,\cdots,p\\ \lambda_j \geqslant 0, & j = 1,2,\cdots,n.\end{cases}$$

Definition 2. If any one of the optimal solutions of (M6) satisfies $V=1$, then DMU_{j0} is stochastically DEA efficient to the indicated levels of probability.

Theorem 2. The optimal value of (M5) $V \leqslant 1$, and if $V=1$, then $\theta_i = 1 (i=1,2,\cdots,m)$ and $\beta_j = 1$ $(j=1,2,\cdots,s)$.

Proof 1. For all the feasible solutions and risk level α, it is true that $0 \leqslant \theta_i \leqslant 1 (i=1,2,\cdots,m), 0 \leqslant \beta_h \leqslant 1 (h=p+1,\cdots,s)$, and $\beta_l \geqslant 1 (l=1,2,\cdots,p)$, then apparently,

$$\frac{1}{m}\sum_{i=1}^{m}\theta_i + \frac{1}{s-p}\sum_{h=p+1}^{s}\beta_h \leqslant 2,$$

$$\frac{1}{p}\sum_{l=1}^{p}\beta_l \geqslant 1,$$

so

$$V \leqslant 1.$$

Proof 2. When $V=1$, we can prove the following situations:

(i) At least there is $\theta_i^* < 1$.

Here, four cases can be thought: At least there are $\beta_l^* > 1$ and $\beta_h^* < 1$; at least there are $\beta_l^* > 1$ and $\beta_h^* = 1$; $\beta_l^* = 1$ and at least there is $\beta_h^* < 1$; and $\beta_l^* = 1$ and $\beta_h^* = 1$.

(ii) At least there is $\beta_l^* > 1$.

Here two cases can be considered: $\theta_i^* = 1$ and at least there is $\beta_h^* < 1$; $\theta_i^* = 1$ and $\beta_h^* = 1$.

(iii) At least there is $\beta_h^* < 1$.

Only one case should exist: $\theta_i^* = 1$ and $\beta_l^* = 1$.

Now we only prove the case that at least there are $\beta_l^* > 1$ and $\beta_h^* < 1$. It is easy to know that

$$\frac{1}{m}\sum_{i=1}^{m}\theta_i + \frac{1}{s-p}\sum_{h=p+1}^{s}\beta_h < 2$$

and

$$\frac{1}{p}\sum_{l=1}^{p}\beta_l > 1,$$

so

$$V < 1,$$

which contradicts $V=1$. Similarly, for all the cases we can conclude either $V>1$ or $V<1$ and converse $V=1$. The proof is completed.

2 An application of the stochastic DEA model with undesirable outputs to the evaluation of anti-HIV therapy

Let T, V represent the concentration of the uninfected $CD4^+$ T cells and free infectious virus particles, respectively, and u_1, u_2 represent two different treatment strategies. As our control classes we choose a measurable function defined on a fixed interval (as treatment cannot be continued for infinite time period due to the side effects) satisfying $0 \leqslant a_i \leqslant u_i(t) \leqslant b_i < 1$ for $i=1,2$. For most of the HIV chemotherapy drugs, the length of the treatment is less than 500 days.

2.1 Finding the decision making units (DMUs)

Deciding the aim of assessment: The aim of assessment is to get more $CD4^+$ T cells and fewer virus particles at a lower cost at different risk levels.

Selecting DMUs: Let the time interval be $[0, T]$ for it must be finite due to hazardous side effects. The interval is divided into n pieces equally, and it is supposed that $0 \leqslant t_1 < t_2 < \cdots < t_n \leqslant T$. DMU_j is defined on the interval $[0, t_j]$.

Selecting inputs and outputs: For the piece number j, let T_j represent the concentration of the uninfected CD4$^+$ T cells, and V_j represent the concentration of the HIV particles, u_{1j} and u_{2j} represent the cost of immune boosting drugs and the cost of viral suppressing drugs respectively. We define

$$U_{1j} = u_{1,1} + u_{1,2} + \cdots + u_{1,j},$$
$$U_{2j} = u_{2,1} + u_{2,2} + \cdots + u_{2,j},$$

where U_{1j} and U_{2j} are inputs, and V_j and T_j are outputs, and V_j is an undesirable output.

Here, in each period, the amount of drug u_1, u_2 remains constant. u_1 takes a value of 0.001, 0.01, $\cdots$, 0.02, and u_2 takes a value of 0.1, 0.2, $\cdots$, 0.9.

Constructing DEA models: Because the changes of inputs are more obvious than the changes of outputs, we should design the DEA models based on inputs according the DEA theory.

2.2 Resource of the data

Before a patient begins his treatment, the doctor should determine the dosage of the drugs. In a sense, it stands for the cost of treatment, hence, these data could serve as deterministic inputs for our study. But the two outputs are stochastic variables, so we need first to obtain the expected value and the standard deviation. In fact, it is difficult to determine $\bar{y}_{rj}$ and b_{rj} because the original clinical data of the AIDS treatment are unavailable. Thus, we have to construct a large sample set by the statistic method. As we can estimate the imprecise status of a patient who is recommended to receive anti-therapy treatment, the data set can be got stochastically according to given status and the differential equation. The classification of the DMUs is given in Table 1 and model data are presented in Table 2.

Table 1. Classification of the DMUs

DMU	Days	u_1	u_2
1	1—5	0.02	0.9
2	6—10	0.02	0.9
3	11—15	0.015	0.7
4	16—20	0.015	0.7
5	21—25	0.02	0.6
6	26—30	0.02	0.6
7	31—35	0.01	0.4
8	36—40	0.005	0.3
9	40—45	0.005	0.2
10	46—50	0.0025	0.1

Table 2. The model data

DMU	U_1	U_2	CD4$^+$ T cells		HIV particles	
			Mean	Standard deviation	Mean	Standard deviation
1	0.02	0.9	425.2675	7.2716	0.9175	0.0030
2	0.04	1.8	474.7678	7.9502	0.9008	0.0025
3	0.055	2.5	515.7582	8.4772	1.0553	0.0045
4	0.07	3.2	559.4640	9.0395	1.0356	0.0040
5	0.09	3.8	621.1698	9.8831	1.1061	0.0049
6	0.11	4.4	688.6345	10.9059	1.0719	0.0051
7	0.12	4.8	725.1409	11.2392	1.2745	0.0075
8	0.125	5.1	744.1901	11.4018	1.3970	0.0090
9	0.13	5.3	763.2992	11.5676	1.5353	0.0114
10	0.1325	5.4	772.7681	11.5912	1.7023	0.0134

2.3 Analysis of efficiency measurement

Assuming that $\sigma = 1$, then

$$\min \frac{1}{2}(\theta_1 + \theta_2) + \beta_2 - \beta_1 - \varepsilon(s_1^+ + s_2^+ + s_3^+ + s_1^-)$$

$$
\text{s.t.}\begin{cases}
\sum_{j=1}^{n}\lambda_j U_{1j} + s_1^+ = \theta_1 U_{1j0} \\
\sum_{j=1}^{n}\lambda_j U_{2j} + s_2^+ = \theta_2 U_{2j0} \\
\sum_{j=1}^{n}\lambda_j [\bar{T}_j + b_{1j}F^{-1}(1-\alpha)] - s_1^- = \beta_1 \bar{T}_{j0} \\
\sum_{j=1}^{n}\lambda_j [\bar{v}_j + b_{2j}F^{-1}(1-\alpha)] + s_3^+ = \beta_2 \bar{T}_{j0} \\
\sum_{j=1}^{n}\lambda_j = 1 \\
s_1^+, s_2^+, s_3^+, s_1^- \geqslant 0 \\
0 \leqslant \theta_i \leqslant 1, \quad i = 1,2 \\
0 \leqslant \beta_2 \leqslant 1 \\
\beta_1 \geqslant 1 \\
\lambda_j \geqslant 0, \quad j = 1,2,\cdots,n.
\end{cases}
$$

Let $\alpha = 0.01$, 0.1, 0.5 or 0.99. The solution of the stochastic model with non-Archimedean infinitesimal is given in Table 3. And we may find that DMU1, DMU2, DMU9 and DMU10 are efficient under all the levels α.

The medical meaning of the results is significant. According to the above models, the anti-HIV therapy is efficient from the beginning to the 10th day, and will last to the 45th day and even to the 50th day. If a patient, to whom the therapy is recommended and he cannot afford the expensive cost of the treatment or he cannot tolerate side effects of drugs, the treatment can stop on the first ten days; otherwise, the treatment should continue to more than 40 days.

Table 3. Stochastic efficiency (%) with different level α under the stochastic DEA model

DMU	$\alpha=0.01$				$\alpha=0.1$			
	θ_1	θ_2	β_1	β_2	θ_1	θ_2	β_1	β_2
1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
3	75.97	70.85	100.00	98.57	75.97	70.86	100.00	98.49
4	77.35	70.81	100.00	88.65	77.36	70.81	100.00	88.62
5	75.15	73.82	100.00	88.12	75.15	73.82	100.00	88.09
6	74.85	77.12	100.00	88.05	74.85	77.12	100.00	88.04
7	81.99	84.07	100.00	93.38	81.99	84.07	100.00	93.33
8	92.73	92.87	100.00	97.63	92.73	92.87	100.00	97.60
9	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\alpha=0.5$				$\alpha=0.99$			
	θ_1	θ_2	β_1	β_2	θ_1	θ_2	β_1	β_2
1	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
2	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
3	75.98	70.86	100.00	98.38	75.97	70.86	100.00	98.49
4	77.36	70.82	100.00	88.59	77.36	70.81	100.00	88.62
5	75.15	73.83	100.00	88.05	75.15	73.82	100.00	88.09
6	74.85	77.12	100.00	88.02	74.85	77.12	100.00	88.04
7	81.99	84.07	100.00	93.33	81.99	84.07	100.00	93.36
8	92.72	92.87	100.00	97.56	92.72	92.87	100.00	97.60
9	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

3 Conclusions

This study proposes a new type of DEA, which incorporates stochastic factors and future information on outputs. To prove its practicality, the proposed DEA model has been applied to the prediction of treatment strategy efficiency for a HIV patient, who is recommended to receive the anti-HIV therapy. And the results provide the patient the optimal strate-

gies of the therapy and information about the time to stop HIV treatment according to his own conditions, such as fitness, economics and so on. Therefore, it has some significant merits in efficiently curing AIDS and decreasing the expenditure of patients.

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